
Contents

Preface	viii
Introduction	1
1 Boolean gaggles	7
1.1 Normal modal logics: K , T , B , $S4$ and $S5$	7
1.2 Kripke-style semantics of the K^* logics	17
1.3 Boolean gaggles: BG s	28
1.4 Semantics for BG s	38
1.5 Problems	48
2 Gaggles: distributive lattices with families of operations	51
2.1 Positive relevance logics: B_+^{ot} , L_+^{ot} and R_+^{ot}	51
2.2 Routley–Meyer-style semantics of the B^* logics	59
2.3 Gaggles and their semantics	71
2.4 Bounded gaggles	84
2.5 Problems	87
3 Nondistributive gaggles	90
3.1 Punctual logics: P , BCI , LL and BCK	90
3.2 Semantics of P^* logics	100
3.3 Nondistributive gaggles: ngGI s	116
3.4 Problems	124

4 Semi-lattice gaggles	126
4.1 Logics with conjunction: $B_{\wedge}^{\circ t}$ and R_{\wedge}°	126
4.2 Logics with disjunction: AC^* and \tilde{RW}_{\vee}^{tt}	133
4.3 Semi-lattice gaggles: sgGls	139
4.4 Problems	149
5 Partial gaggles	152
5.1 Lambek calculi: LQ and LA	152
5.2 Three relevance logics: TW_{\rightarrow} , $E_{\rightarrow}^{\circ t}$ and R_{\rightsquigarrow}^t	159
5.3 Partial gaggles: pgGls	165
5.4 Problems	172
6 Constants	175
6.1 Extensional constants: T and F	176
6.2 Intensional constants: t and f	181
6.3 Combinatory constants	184
6.4 Models of structurally free logics	188
6.5 Problems	195
7 Inductively defined operations	198
7.1 Kleene logic and action logic with converse	199
7.2 Action logic with conjunction	208
7.3 Distributive action logic	212
7.4 Relation algebras with star	215
7.5 Problems	218
8 Canonicity	220
8.1 Modal logics, BAOs and BGs	220
8.2 Canonicity generalized	228
8.3 Distributive and other gaggles	235
8.4 Problems	239
9 Duality	242
9.1 Topologies and categories	244
9.2 Boolean algebras and BGs	252
9.3 Distributive lattices and gaggles	262
9.4 Lattices and ngGls	270
9.5 Posets and pgGls	283
9.6 Problems	286

10 Heterogeneous topics	289
10.1 Modal gaggles	289
10.2 Gaggle semantics for intuitionistic logic	291
10.3 Possibility and necessity linked without negation	295
10.4 Operational semantics for <i>BCI</i>	298
10.5 Boolean algebras with nonnormal fusion \circ	301
10.6 Boolean algebras with normal monotone \bullet	303
10.7 Galois logics	305
10.8 Problems	312
Appendix	314
A Ordered sets	315
A.1 Types of order relations	315
A.2 Bounds and extremal elements	318
A.3 Cones, cocones and closure operations	320
A.4 Separation in posets	326
A.5 Zorn's lemma	327
B Semi-lattices and lattices	328
B.1 Algebras	328
B.2 Meet and join semi-lattices	335
B.3 Lattices	337
B.4 Filters and ideals	340
B.5 Representations of (semi-)lattices	343
B.6 Separation in (semi-)lattices	347
C Distributive lattices and Boolean algebras	350
C.1 Distributive lattices	350
C.2 De Morgan lattices and ortholattices	352
C.3 Boolean algebras	354
C.4 Representations	357
C.5 Separation in distributive lattices	359
Bibliography	362
Index	371
Table of symbols	380